4.4.1. Applying the equation of momentum in the horizontal direction,
\[ \Sigma F_x = \rho Q (V_{s_2} - V_{s_1}) \]
-200 = 1000(\pi/4)(0.03)^2 V_{s_1} (0 - V_{s_1}) = -0.707 V_{s_1}^2
\[ V_{s_1} = 16.82 \text{ m/s.} \]
\[ Q = (\pi/4)(0.03)^2(16.82) = 0.012 \text{ m}^3/\text{s.} \]

\[ \frac{P_1}{\gamma} + z_1 + \frac{V_{1}^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_{2}^2}{2g} \]
\[ 0 + 0 + \frac{V_{1}^2}{2(32.2)} = 0 + 150 + 0 \]
\[ V_1 = \sqrt{2(32.2)(150)} = 98.285 \text{ ft/s.} \]
4.5.3. The energy equation is applied between points 1 and 2:

\[ \frac{P_1 + V_{A_1}^2}{2g} + \frac{V_{C_1}^2}{2g} + 0 = \frac{P_2 + V_{A_2}^2}{2g} + 0 + f_A \frac{L_A}{D_A} \frac{V_{A_1}^2}{2g} + \frac{(V_{A_1} - V_{A_2})^2}{2g} + f_B \frac{L_B}{D_B} \frac{V_{A_2}^2}{2g} + f_c \frac{L_C}{D_C} \frac{V_{C_1}^2}{2g} + \]

\[ f_c \frac{L_C}{D_C} \frac{V_{C_2}^2}{2g} \]

From continuity, \( V_B = \frac{D_A^2}{D_B^2} \cdot V_A = \frac{(4/12)^2}{(8/12)^2} \cdot V_A = \frac{4}{9} V_A \). Hence,

\[ V_{C_1} = \frac{D_A^2}{D_C^2} \cdot V_A = \frac{(4/12)^2}{(6/12)^2} \cdot V_A = \frac{4}{9} V_A \cdot \frac{2}{2} \]

\[ 900 \ \frac{V_{A_1}^2}{2g} + (V_{A_1} - V_{A_2})^2 \]

\[ f_B \frac{750}{(8/12)^2} \left( \frac{(9/8)^2}{4} \right)^2 + f_c \frac{1200}{(6/12)^2} \left( \frac{(9/8)^2}{4} \right)^2 \]

Simplifying this equation and substituting for \( p_1 - p_2 \) gives

\[ \frac{15(144)}{62.4} = \frac{V_{A_1}^2}{2(32.2)} \]

\[ 2229.231 = V_{A_1}^2 \left( 2700 f_a + 70.313 f_b + 474.074 f_c + 0.230 \right) \]

\[ V_{A_1} = \frac{2700 f_a + 70.313 f_b + 474.074 f_c - 0.230}{2(32.2)} \]

Assuming the flow to be in the complete turbulence zone,

\[ f_a = 0.023, f_b = 0.019, f_c = 0.020 \]

Thus,

\[ 2229.231 = V_{A_1}^2 \left[ 2700(0.023) + 70.313(0.019) + 474.074(0.020) - 0.230 \right] \]

\[ V_{A_1} = 5.538 \text{ ft/s} \]

\[ V_{A_2} = \frac{V_{A_1}}{4} = \frac{5.538}{4} = 1.385 \text{ ft/s} \]

\[ V_{C_1} = \frac{V_{A_1}}{4} = \frac{5.538}{4} = 2.461 \text{ ft/s} \]

\[ R_a = \frac{V_{A_1}}{\nu} = \frac{5.538}{1.059 \times 10^{-5}} = 1.7 \times 10^5 \]

\[ R_b = \frac{V_{A_1}}{\nu} = \frac{1.385}{1.059 \times 10^{-5}} = 8.7 \times 10^4 \]

\[ R_c = \frac{V_{A_1}}{\nu} = \frac{2.461}{1.059 \times 10^{-5}} = 1.2 \times 10^5 \]

Check for f values from Moody diagram: \( f_a = 0.0235, f_b = 0.022, \) and \( f_c = 0.022 \)

Thus, \( 2229.231 = V_{A_1}^2 \left[ 2700(0.0235) + 70.313(0.022) + 474.074(0.022) - 0.230 \right] \]

\[ V_{A_1} = \frac{2700 f_a + 70.313 f_b + 474.074 f_c - 0.230}{2(32.2)} \]

\[ 2229.231 = V_{A_1}^2 \left[ 2700(0.0235) + 70.313(0.022) + 474.074(0.022) - 0.230 \right] \]

\[ V_{A_1} = 5.445 \text{ ft/s} \]

\[ V_{A_2} = \frac{V_{A_1}}{4} = \frac{5.445}{4} = 1.361 \text{ ft/s, and} \]

\[ V_{C_1} = \frac{V_{A_1}}{4} = \frac{5.445}{4} = 2.420 \text{ ft/s.} \]

\[ R_a = \frac{V_{A_1}}{\nu} = \frac{5.445(1.059 \times 10^{-5})}{1.059 \times 10^{-5}} = 1.7 \times 10^5 \]

\[ R_b = \frac{V_{A_1}}{\nu} = \frac{1.361(1.059 \times 10^{-5})}{1.059 \times 10^{-5}} = 8.6 \times 10^4 \]

\[ R_c = \frac{V_{A_1}}{\nu} = \frac{2.420(1.059 \times 10^{-5})}{1.059 \times 10^{-5}} = 1.1 \times 10^5 \]

Check for f values: Referring to Moody diagram, f values are similar to the

previous f values obtained for all pipes. Thus,

\[ V_{A_1} = 5.445 \text{ ft/s} \]

\[ Q = V_{A_1}A_1 = \frac{\pi}{4}(4/12)(5.445) = 0.475 \text{ ft}^3/\text{s} = 213.254 \text{ gpm.} \]
**Additional Problem # 1:**

Given data:

Material of pipe: Cast iron, Diameter of pipe= 15 cm

Length of pipe = 200m

Solution:

\[ \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{L_f} + \Sigma h_{Lm} \]

\[ 0 + 0 + 35 = 0 + \frac{V_2^2}{2g} + 2 + h_{L_f} + \Sigma h_{Lm} \] \[ \text{.........................................................(1)} \]

\[ h_{L_f} = f L \frac{V_2^2}{2gd} = \left( f \times 200 \times V_2^2 \right) / \left( 2 \times 9.81 \times 0.15 \right) \]

\[ h_{L_f} = 68 f V_2^2 \]

\[ K_s \text{ for cast iron} = 0.00026 \text{ m} \]

\[ K_s / D = 0.00173, \]

Therefore, from Moody’s diagram \( f = 0.023 \)

\[ h_{L_f} = 68 \times 0.023 \times V_2^2 = 1.564 V_2^2 \]

\[ \Sigma h_{Lm} = h_{L\text{entrance}} + h_{L\text{valve}} + h_{L\text{exit}} + h_{L\text{pipeline}} \]

\[ \Sigma h_{Lm} = K_e \frac{V_2^2}{2g} + K_{\text{valve}} \frac{V_2^2}{2g} + V_2^2/2g + h_{L\text{pipeline}} \]

\[ h_{L\text{pipeline}} = \text{Neglected} \]

\[ \Sigma h_{Lm} = (0.5)V_2^2/2g + (0.07) V_2^2/2g + V_2^2/2g \]

\[ \Sigma h_{Lm} = (0.5+0.14 + 1) V_2^2/2g \quad \text{[K_{valve} = 0.14(assumed to be fully open)]} \]

\[ \Sigma h_{Lm} = 1.64 V_2^2/2g \]

From (1);

\[ 0 + 0 + 35 = 0 + \frac{V_2^2}{2g} + 2 + h_{L_f} + \Sigma h_{Lm} \]

\[ 35-2 = V_2^2/2g + 1.564 V_2^2 + 1.64 V_2^2/2g \]

\[ V_2 = 4.41 \text{ m/sec} \]
\[ h_{L_f} = 1.564 \, V_2^2 = 1.564 \times (4.41)^2 = 30.42 \, m \]

\[ \Sigma h_{L_m} = 1.64 \, V_2^2 / 2g = 1.62 \, m \]

a) Refer Figure –I

b) \[ Q = A \, V = \left( \frac{\pi}{4} \right) d_2^2 \times V_2 = \left( \frac{\pi}{4} \right) (0.15)^2 \times 4.41 \]
   \[ Q = 0.078 \, m^3/sec \]

c) \[ A_2 = \left( \frac{\pi}{4} \right) d_2^2; \quad A_3 = \left( \frac{\pi}{4} \right) d_3^2 \]
   \[ d_3 = 0.05 \, m; \quad d_2 = 0.15 \, m \]

From continuity equation,
   \[ A_3 \, V_3 = A_2 \, V_2 \]
   \[ (\pi / 4) \times (0.05)^2 \times V_3 = (\pi / 4) \times (0.15)^2 \times 4.41 \]
   \[ V_3 = 39.69 \, m/sec \]
   \[ Q = A \, V = \left( \frac{\pi}{4} \right) d_3^2 \times V_3 = \left( \frac{\pi}{4} \right) (0.05)^2 \times (39.69) \]
   \[ Q = 0.078 \, m^3/sec \]

**FIGURE - I**
Additional Problem # 2:

Given data:

Diameter of pipe: 2 inch
Pressure in water main: 60 psi
Length of service pipe: 60’
Flow rate in the pipe: 15 gpm
Height of the 3rd floor: 55’
Temperature: 70°F

From Table T 2.1.1 we have,

Specific weight of water (γ) = 62.30 lb/ft³
Kinematic viscosity = 1.059 x 10⁻⁶ ft²/sec

Velocity in the pipe (V) = Q/V = 15 / (π/ 4) (2/12)² = 1.53 ft/sec

Finding head loss due to friction = hf = f L V²/2gd
For finding the friction factor use Moody’s diagram

We have Reynolds number = Re = VD/γ = 24,000

From Moody’s diagram, Ks = 0.0005’

Ks / D = 0.003,

From Moody’s diagram, the turbulent flow and for Ks / D = 0.003 we have friction factor
f=0.0265

hf = 0.353’

P₁/γ + V₁²/2g + Z₁ = P₂/γ + V₂²/2g + Z₂ + hf

In which V₁ = V₂ and Z₁ =0 and Z₂=55’

Hence P₂=36.0 psi

Result: The pressure at the main will be sufficient to meet the shower requirements.